Production of bound triplet $\mu^+\mu^-$ system in collisions of electrons with atoms

N. Arteaga-Romero[(a)], C. Carimalo[(b)], and V.G. Serbo[(b,c)]

[(a)] *Université Paris VI, 4 place Jussieu, Paris Cedex 05 France*

[(b)] *Laboratoire de Physique Nucléaire et de Hautes Energies, IN2P3-CNRS*

Université Paris VI & Paris VII, 4 place Jussieu, Tour 33, F-75252, Paris Cedex 05 France

e-mail : carimalo@in2p3.fr

[(c)] *Novosibirsk State University, Physics Department, Novosibirsk, 630090 Russia*

e-mail : serbo@math.nsc.ru

(February 1, 2008)

PACS number(s): 12.38.-t, 36.10.Dr, 13.60.-r

Abstract

This paper deals with the production of orthodimuonium (OM) ($\mu^+\mu^-$ atom in triplet state) in collisions of high-energy electrons with nuclei or atoms. This reaction was previously studied by Holvik and Olsen [Phys. Rev. D **35**, 2124 (1987)] on the basis of a bremsstrahlung mechanism where OM is produced by only one virtual photon. In the present paper we consider a competing three-photon mechanism where the production of OM results from the collision of a photon generated by the electron with two photons emitted by the nucleus. We derive the corresponding energy spectrum and production rate of OM and show that the three-photon mechanism is the dominant one for heavy atom target.

I. INTRODUCTION

It is known that $\mu^+\mu^-$ atoms exist in two spin states: paradimuonium (PM, singlet state $n\,^1S_0$) with lifetime in the ground state $\tau_0 = 0.6 \cdot 10^{-12}$ s and orthodimuonium (OM, triplet state $n\,^3S_1$) with lifetime in the ground state $\tau_1 = 1.8 \cdot 10^{-12}$ s. The dominant decay processes are

$$\mathrm{PM} \to \gamma\gamma, \quad \mathrm{OM} \to e^+e^- \,. \tag{1}$$

The main physical motivation to study dimuonium production lies in the fact that dimuonium is one of the simplest hydrogenlike atom, that is very convenient for testing fundamental laws. Up to now there is a lot of theoretical predictions on dimuonium properties (for a review see, for example, proceedings [1]) but dimuonium has not been observed yet.

A promising method to create dimuonium — its production at relativistic heavy ion colliders — has been recently considered in Ref. [2]. Another attractive possibility is the electroproduction of dimuonium on atoms. The

PM electroproduction cross section was calculated in Ref. [3] in the main logarithmic approximation, and discussed in more detail in Ref. [4]. In these two papers a two-photon mechanism was considered where the incident electron and the nucleus emit each virtual photons which then collide to produce a C-even PM state.

In Ref. [4] the electroproduction of C-odd OM state on atoms was assumed to proceed from the bremsstrahlung mechanism of Fig. 1. The obtained spectrum of OM production on nucleus of charge Ze is

$$\frac{d\sigma_{\rm br}(x)}{dx} = \sigma_0 \, f_{\rm br}(x), \quad x = \frac{E_{\mu\mu}}{E_e}\,, \quad \sigma_0 = \frac{\zeta(3)}{4}\,\frac{\alpha^5 (Z\alpha)^2}{m_\mu^2}\,, \tag{2}$$

$$f_{\rm br}(x) = \frac{x(1-x)}{(1-x+\varepsilon)^2}\left(1 - x + \frac{1}{3}x^2\right)\left[\ln\frac{(1-x)^2 E_e^2}{(1-x+\varepsilon)m_\mu^2} - 1\right]\,, \quad \varepsilon = \frac{m_e^2}{4m_\mu^2}$$

where E_e and $E_{\mu\mu}$ are the energies of electron and OM respectively, and m_e and m_μ are the electron mass and the muon mass respectively; $\alpha \approx 1/137$ and $\zeta(3) = 1.202...$. The obtained cross section has the form

$$\sigma_{\rm br} = 7.16\,\sigma_0\,(L - 2.73)\,, \quad L = \ln\frac{E_e}{2m_\mu} \tag{3}$$

(note that this cross section is positive only for $E_e > 3.2$ GeV).

However, the analysis of Ref. [4] is incomplete since it did not take into account the important three-photon mechanism depicted in Fig. 2. Our aim in the present paper is to improve this previous analysis by estimating the production rate provided by the latter mechanism and by pointing out some of its main features. As we shall show, the three-photon process competes with the bremsstrahlung one and even predominates in the case of high-energy electron scattering by heavy atom target.

II. CALCULATION OF THREE-PHOTON ORTHODIMUONIUM PRODUCTION

At high electron energy ($E_e \gg m_\mu$) the cross section $d\sigma_{3\gamma}$ corresponding to the diagram of Fig. 2 can be calculated using the equivalent-photon approximation. In this approximation the cross section $d\sigma_{3\gamma}$ is expressed as the product of the number of equivalent photons $dn_\gamma(x, Q^2)$ generated by the electron, by the cross section $d\sigma_{\gamma Z}$ for the real photoproduction of OM on the nucleus

$$d\sigma_{3\gamma} = dn_\gamma \, \sigma_{\gamma Z} \, , \quad dn_\gamma(x, Q^2) = \frac{\alpha}{\pi} \frac{dx}{x} \frac{dQ^2}{Q^2} \left[1 - x + \frac{1}{2}x^2 - (1-x) \frac{Q^2_{\min}}{Q^2} \right] ,$$

$$Q^2 = -q^2 \, , \quad Q^2_{\min} = \frac{x^2 m_e^2}{1-x} \, . \tag{4}$$

After integration of $dn_\gamma(x, Q^2)$ over Q^2 from $Q^2_{\min}$ up to $Q^2 \sim m_\mu^2$ we obtain the energy spectrum of equivalent photons

$$\frac{dn_\gamma(x)}{dx} = \frac{\alpha}{\pi} \frac{1}{x} \left[\left(1 - x + \frac{1}{2}x^2 \right) \ln \frac{(1-x) m_\mu^2}{x^2 m_e^2} - 1 + x \right] . \tag{5}$$

The accuracy of this expression is a logarithmic one, i.e. the omitted items are of the order of $1/l \sim 1/15$ where $l = \ln\left[(1-x)m_\mu^2/(x^2 m_e^2)\right]$.

The photoproduction cross section $\sigma_{\gamma Z}$ can be found in Ref. [2]. At high photon energy ($xE_e \gg m_\mu$), the corresponding amplitude for photoproduction of OM in $n\,{}^3S_1$ state on nucleus of mass M_Z takes the form

$$M_{\gamma Z} = 4i\alpha^4 Z^2 \frac{2xE_e M_Z}{n^{1/3}} \int \frac{F(\mathbf{k}^2_{1\perp})\, F(\mathbf{k}^2_{2\perp})}{\mathbf{k}^2_{1\perp}\, \mathbf{k}^2_{2\perp}} \; \mathbf{e}_\gamma \, . \mathbf{e}^*_{\mathrm{OM}} \;\times \tag{6}$$

$$\left[\frac{4m_\mu^2}{4m_\mu^2 + \mathbf{p}_\perp^2} - \frac{4m_\mu^2}{4m_\mu^2 + (\mathbf{p}_\perp - 2\mathbf{k}_{1\perp})^2} \right] \delta(\mathbf{k}_{1\perp} + \mathbf{k}_{2\perp} - \mathbf{p}_\perp)\, d^2\mathbf{k}_{1\perp}\, d^2\mathbf{k}_{2\perp}$$

where $F(\mathbf{k}^2_{i\perp})$ is the nucleus form factor, $\mathbf{p}$ is the momentum of OM, $\mathbf{e}_\gamma$ and $\mathbf{e}_{\mathrm{OM}}$ are the polarization vectors for the equivalent photon and the final OM respectively. Because of the rapid decrease of the nuclear form factor at large transverse momenta, the main contribution to the amplitude comes from values of tranverse momenta such that

$$\mathbf{k}_{i\perp}^2 \lesssim \frac{1}{\langle r^2 \rangle} \tag{7}$$

where $\langle r^2 \rangle$ is the mean square radius of the charge distribution of the nucleus. We use below the parameter Λ defined as

$$\frac{1}{6}\langle r^2 \rangle = \frac{1}{\Lambda^2}\,, \quad \Lambda = \frac{405}{A^{1/3}}\ \text{MeV} \tag{8}$$

where A is the atomic mass number, $\Lambda = 70$ MeV for Pb and $\Lambda = 120$ MeV for Ca. The cross section $\sigma_{\gamma Z}$ is found to be a constant at high photon energy $xE_e \gg m_\mu$. It was calculated in Ref. [2] by numerical integration of Eq. (6), using a realistic nuclear form factor. After summing up over all $n\,^3S_1$ states of OM, the final result is

$$\sigma_{\gamma Z} = 4\pi\alpha\sigma_0\, B \left(\frac{Z\Lambda}{m_\mu}\right)^2 , \quad B = 0.85\,. \tag{9}$$

As a result, the shape of the energy spectrum of OM is just that of the equivalent photon spectrum :

$$\frac{d\sigma_{3\gamma}(x)}{dx} = \sigma_0\, f_{3\gamma}(x)\,, \tag{10}$$

$$f_{3\gamma}(x) = \left(\frac{Z\alpha\Lambda}{m_\mu}\right)^2 \frac{4B}{x}\left[\left(1 - x + \frac{1}{2}x^2\right)\ln\frac{(1-x)m_\mu^2}{x^2 m_e^2} - 1 + x\right].$$

The total cross section is obtained after integration over x (from $x_{\min} = 2m_\mu/E_e$ up to $x_{\max} = 1$)

$$\sigma_{3\gamma} = 4B\,\sigma_0 \left(\frac{Z\alpha\Lambda}{m_\mu}\right)^2 \left[L^2 + \left(L - \frac{3}{4}\right)\ln\frac{m_\mu^2}{m_e^2} - L - \frac{\pi^2}{6} - \frac{1}{8}\right] =$$

$$= 3.4 \left(\frac{Z\alpha\Lambda}{m_\mu}\right)^2 \sigma_0 \left[L^2 + 9.7\,(L-1)\right]. \tag{11}$$

III. RESULTS AND CONCLUSIONS

1. Let us compare the energy and angular distributions corresponding to the bremsstrahlung and to the three-photon production of OM respectively. The energy spectra are given by Eqs. (2), (10) or by the curves for $f_{\text{br}}(x)$

and $f_{3\gamma}(x)$ presented in Fig. 3. The bremsstrahlung function $f_{\rm br}(x)$ has a peak at large x ($x \approx 1$); it depends on the electron energy but not on the target properties. On the other hand, the three-photon function $f_{3\gamma}(x)$ has a peak at small x; it does not depend on the electron energy but it strongly depends on the type of nucleus.

The respective angular distributions are also different. The typical emission angle of OM in bremsstrahlung production was estimated in Ref. [4] as

$$\theta_{\rm br} \sim \frac{m_\mu}{E_e} \tag{12}$$

The analogous typical angle in three-photon production is of the order of (see Eq. (6))

$$\theta_{3\gamma} \sim \frac{\Lambda}{x E_e} \tag{13}$$

and is thus much larger than $\theta_{\rm br}$ in the range $x \ll 1$, where the three-photon process dominates.

Taking into account the quite different energy and angular distributions of OM in the two production mechanisms, we can conclude that the interference between bremsstrahlung and three-photon productions should be very small.

2. The total cross sections for the production mechanisms here discussed are given by Eqs. (3), (11) and are presented as functions of the electron energy in Figs. 4 and 5, for Pb target and Ca target respectively. It is clearly seen that the bremsstrahlung mechanism is the most important for electroproduction on light nuclei while the three-photon mechanism is dominant for electroproduction on heavy nuclei.

The knowledge of the total cross section gives us a possibility to estimate the production rate. A detailed procedure to obtain such a number can be found in Ref. [4] where the rate for bremsstrahlung production of OM was estimated. For example, this rate in the case of Pb target is about 3 orthodimuonia per minute for an electron energy $E_e = 10$ GeV and electron current of 1 mA. According to Fig. 4 the rate provided by the three-photon orthodimuonium production for the same example is 2.5 times larger — about 7 orthodimuonia per minute.

3. Thorought this paper we have considered the electroproduction of OM on nuclei. Let us briefly discuss the electroproduction of OM on atoms where a possible atomic screening has to be taken into account. It is known that the atomic screening becomes important when the minimal momentum transfered to atom $2m_\mu^2/E_{\mu\mu} = 2m_\mu^2/(xE_e)$ becomes comparable with the typical atomic momenta $\sim m_e\alpha Z^{1/3}$. In other words, the atomic screening should be taken into account when the electron energy E_e becomes of the order of or larger than the characteristic energy

$$\frac{2m_\mu^2}{m_e\alpha Z^{1/3}} = \frac{6000}{Z^{1/3}}\ \text{GeV}\,. \tag{14}$$

4. In the present paper we have calculated three-photon production of OM due to diagram of Fig. 2 with two photons being exchanged with the nucleus. But the C-odd $\mu^+\mu^-$ bound system can also be produced in collision of a photon generated by the electron, with 4, 6, 8, ... photons exchanged with the target. At first sight, this leads to an additional factor $(Z\alpha)^{n-2}$ for n exchanged photons. This is just the case for electroproduction of orthopositronium on nuclei. Since for heavy nuclei the parameter $Z\alpha$ is not small (for example, $Z\alpha = 0.6$ for Pb), the whole series in $(Z\alpha)^2$ has to be summed. According to Ref. [5], the high-order $(Z\alpha)^2$ effects decrease the orthopositronium production cross section by about 40 % for the Pb target. However, for OM production we should take into account the restriction

of the transverse momenta $k_{1\perp}$, $k_{2\perp}, ...,$ $k_{n\perp}$ due to nuclear form factors on the level given by Eq. (7). As a result, the effective parameter of the perturbation theory is not $(Z\alpha)^2$ but

$$\frac{(Z\alpha)^2}{\langle r^2 \rangle\, m_\mu^2} < 0.03 \qquad (15)$$

and, therefore, we can restrict ourselves to three-photon production only.

ACKNOWLEDGMENTS

The authors wish to thank Prof. I. F. Ginzburg for useful discussions. V.G. Serbo is grateful to University Paris VI for a grant that allowed his stay in Paris where this work was completed, and to the Laboratoire de Physique Nucléaire et de Hautes Energies (LPNHE) for warm hospitality.

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FIGURES

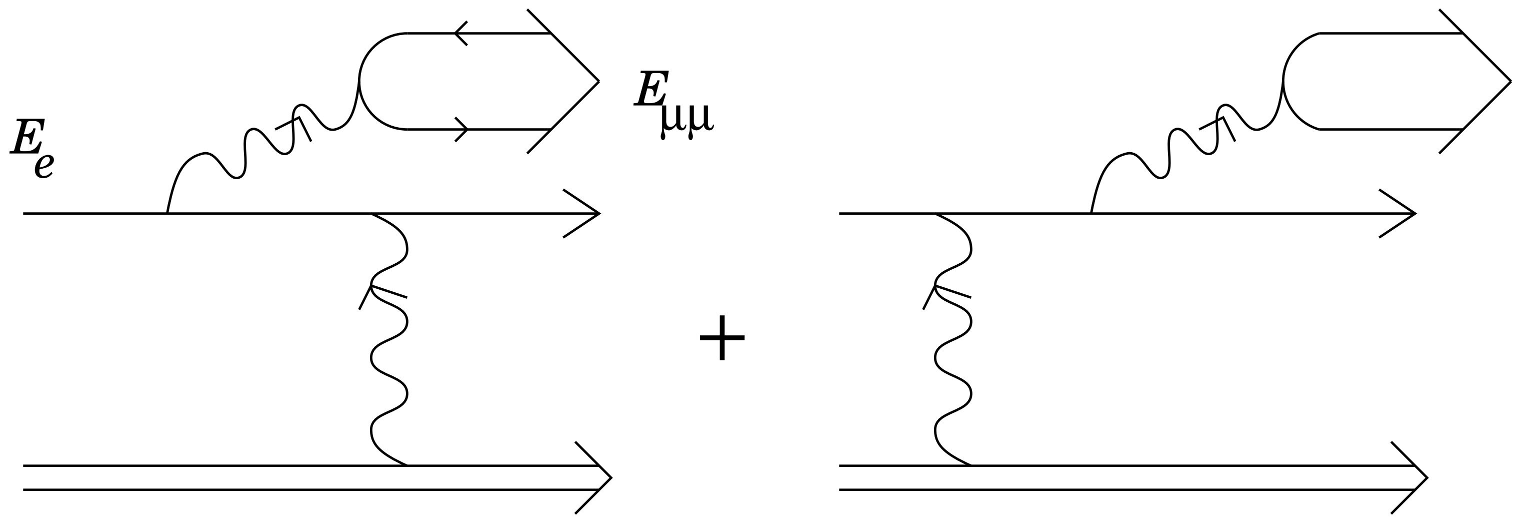
FIG. 1. Bremsstrahlung production of orthodimuonium on nucleus

FIG. 2. Three-photon production of orthodimuonium on nucleus

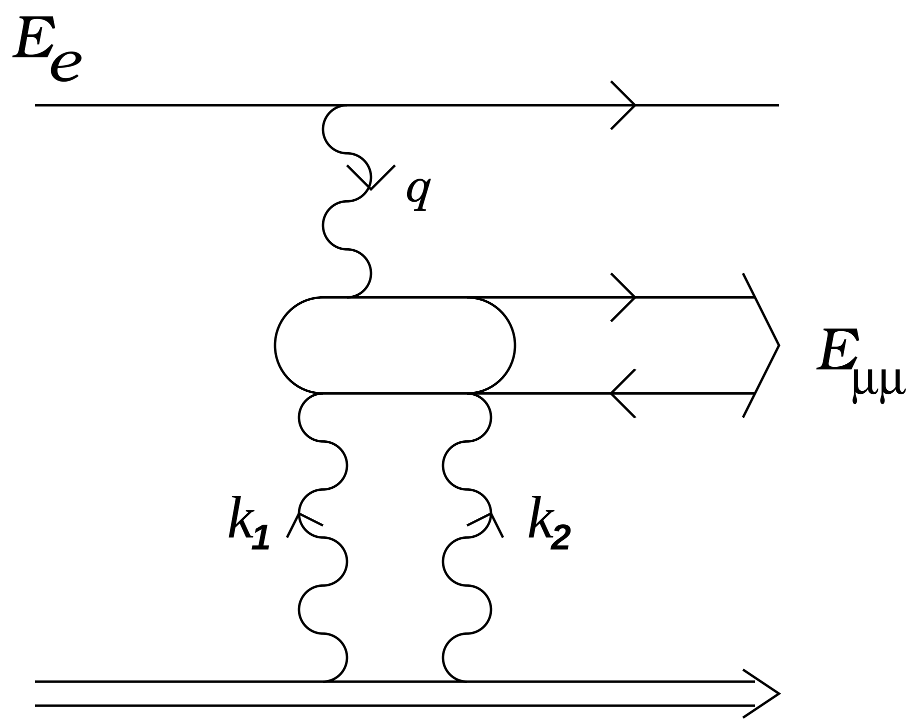
FIG. 3. The orthodimuonium spectra $(1/\sigma_0)\,(d\sigma/dx)$ with $\sigma_0 = 0.3\,\alpha^5(Z\alpha)^2/m_\mu^2$ for three-photon production on Pb (solid line) and Ca (dashed line); for bremsstrahlung production at $E_e = 100$ GeV (dot-dashed line), at $E_e = 10$ GeV (dot-long-dashed line) and at $E_e = 5$ GeV (dotted line)

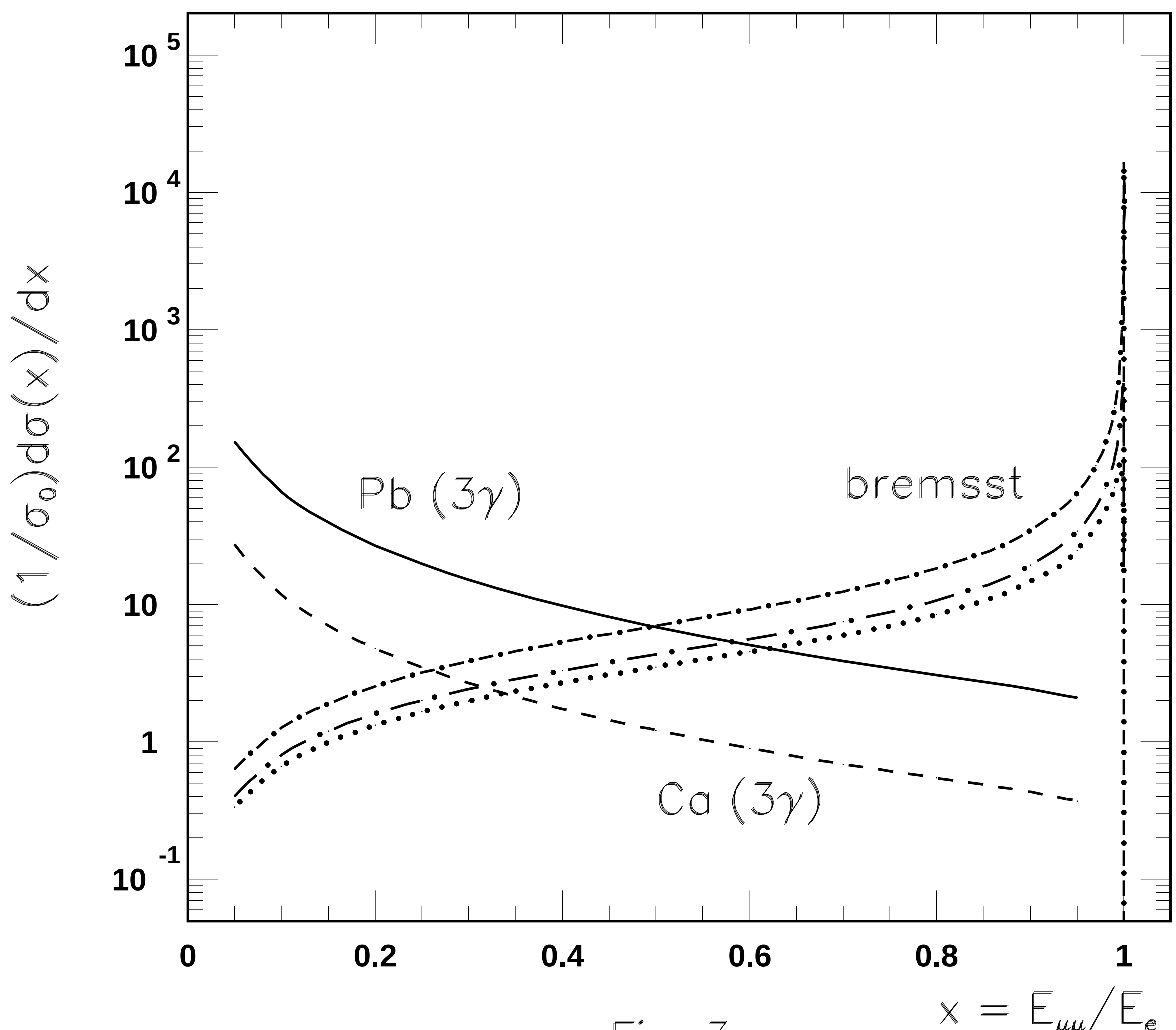
FIG. 4. The total cross section for three-photon (solid line) and bremsstrahlung (dashed line) production of orthodimuonium on Pb

FIG. 5. The same as in Fig. 4 but for Ca target



E_e
$E_{\mu\mu}$
+


E_e
q
$E_{\mu\mu}$
k_1
k_2



$(1/\sigma_0)d\sigma(x)/dx$
10^5
10^4
10^3
10^2
10
1
10^{-1}
Pb (3γ)
bremsst
Ca (3γ)
0
0.2
0.4
0.6
0.8
1
$x = E_{\mu\mu}/E_e$

Fig. 3

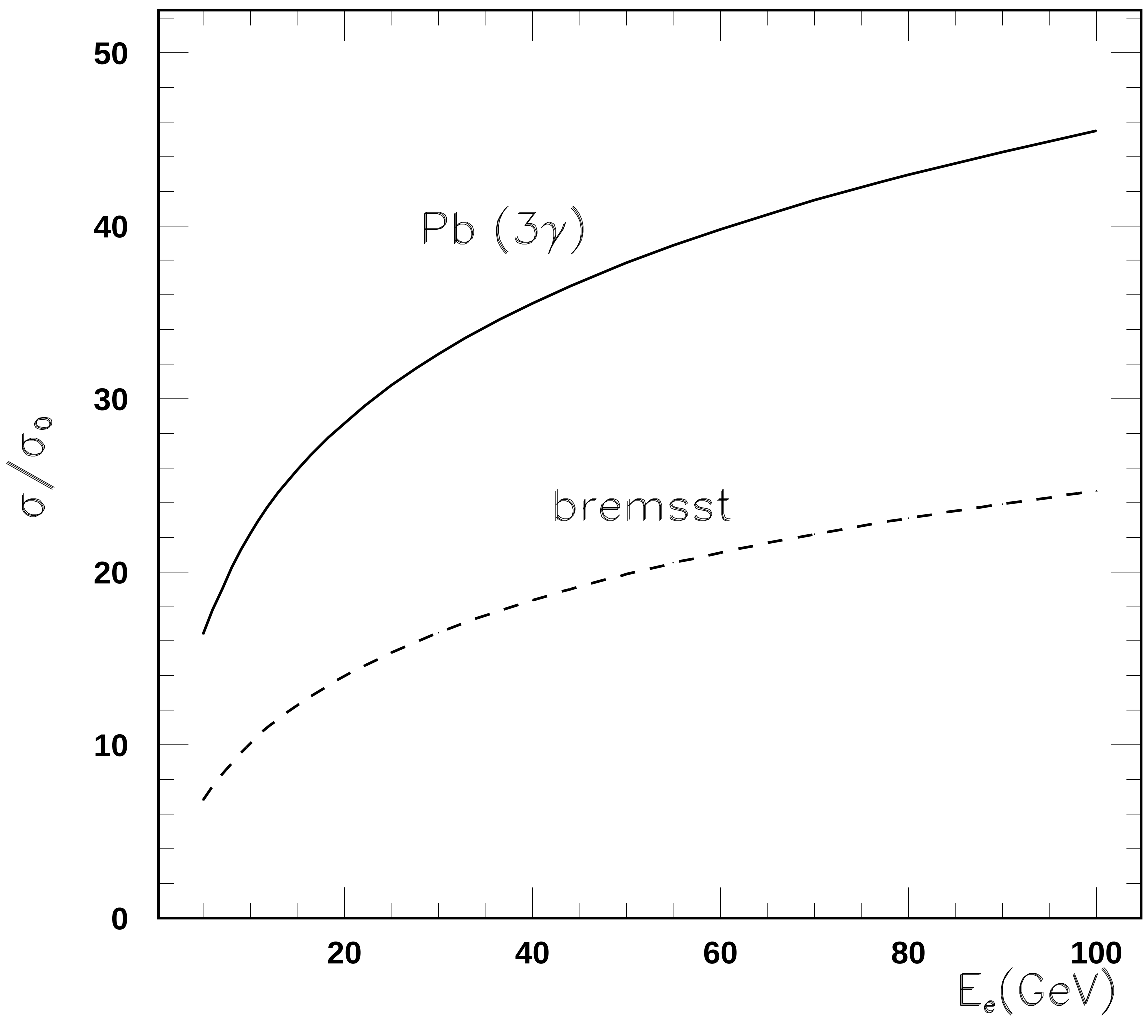


Fig. 4

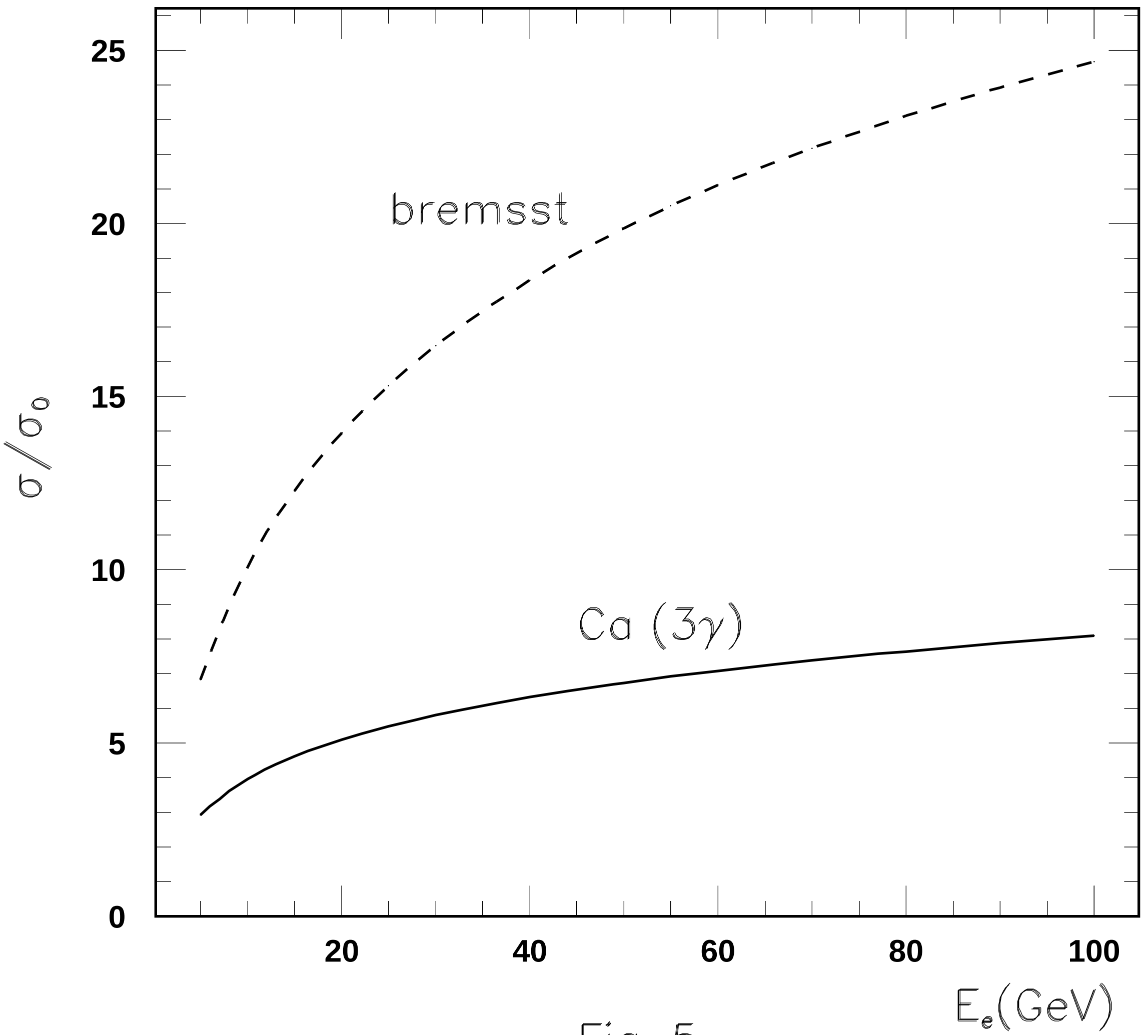


Fig. 5